Analysis including reliability, income and cost for Power systems

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***Abstract*—This paper presents a mathematical method for analysis of reliability and associated cost regarding electrical power grids and similar systems. The reliability analysis is based on Markov models established by means of unit models, which is an efficient method for analysis of large systems. General principles concerning application of the method are presented and illustrated by an example from a local power grid in Norway.**

***Index Terms*—Reliability analysis, Markov models, Power systems, Economic models**

I. Introduction

In recent years the increasing economic requirements concerning electric power make it important to identify and document the reliability for power systems and the associated expected economic consequences. This can be done using reliability analysis combined with power flow models and economic models. Typical strategies that can be analyzed include operation, maintenance and investments.

The probability analysis to be applied is based on a mathematical method that composes Markov models by means of unit models [1], [2]. This method makes analysis of large systems possible without affecting the degree of details. The basic theory used in development of the method is established on the general Markov probability theory [3], [4] and the excellent theory concerning power systems in [5].

The main purpose of the project that constitutes the basis for this paper was to demonstrate that Markov models could be used in calculation of practical power system reliability.

II. Electrical Power Transmission Models

A. Balance equations and cost function

Figure 1 shows a typical power transmission network. This network will be used in the analysis. Branch 1, 2 and 3 represents power lines with generators, branch 4 a transformer and branch 5 a power line. The branch capacity limitations are shown in table 1. This example is a real power grid at the West Coast of Norway.

The unit models are shown in table 2. The total number of unit models is 17, 10 models with 3 states and 7 models with 2 states. The states are:

Power transmission
{Functioning, Temporary failure, Permanent failure}
Protection
{Functioning, Failed, Erroneous operation}
Auto re-closing and common mode power supply
{Functioning, Failed}

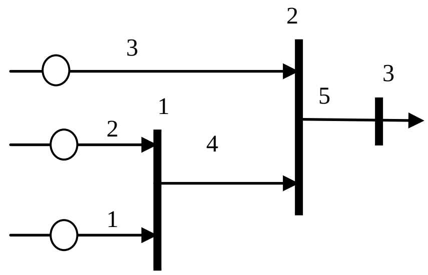

Fig. 1. Typical power transmission network.

Branch	Description	Maximal power flow [MW]
1	Line 1	68
2	Line 2	82
3	Line 3	10
4	Transformer 132/22 kV	25
5	Load point line	22

Table 1. Branch capacity limitations.

Unit models		**Branch no.**				
Component	**No. of states**	1	2	3	4	5
Power transmission	3	x	x	x	x	x
Protection	3	x	x	x	x	x
Auto re-closing	2	x	x			
Common mode power supply	2	x	x	x	x	x

Table 2. Unit models used in the branches.

The transmission capability in the network is restricted by the capacity limits. As a result the network will have several function levels depending on the network failure status, the power flow limitations, and the demand.

Fictive generators injecting power at the load nodes can be used to model power not delivered. By giving these generators extremely high production cost, they will only be in production if the real power supply to the load nodes is in a shortage situation.

The power balance equations and the cost function can be formulated as:

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$$\begin{pmatrix}\mathbf{M}_G & \mathbf{M}_T & \mathbf{M}_F\end{pmatrix}\begin{pmatrix}\mathbf{P}_G\\ \mathbf{P}_T\\ \mathbf{P}_F\end{pmatrix} = \mathbf{P}_D \qquad (1)$$

$$J = \begin{pmatrix}\mathbf{c}_G & \mathbf{c}_T & \mathbf{c}_F\end{pmatrix}\begin{pmatrix}\mathbf{P}_G\\ \mathbf{P}_T\\ \mathbf{P}_F\end{pmatrix} \qquad (2)$$

where

- $\mathbf{M}_G$: Network structure matrix for generator branches
- $\mathbf{M}_T$: Network structure matrix for transmission branches
- $\mathbf{M}_F$: Network structure matrix for fictive generators
- $\mathbf{P}_G$: Generated power [kW]
- $\mathbf{P}_T$: Transmitted power [kW]
- $\mathbf{P}_F$: Fictive power generation [kW]
- $\mathbf{P}_D$: Power demand [kW]
- J : Operation cost [NOK]
- $\mathbf{c}_G$: Specific generation cost [NOK/kW·h]
- $\mathbf{c}_T$: Specific transmission cost [NOK/kW·h]
- $\mathbf{c}_F$: Specific fictive generation cost [NOK/kW·h].

The elements in the structure matrix are:

$$M_{ij} = \begin{cases} 1 & \text{if branch no. i goes to node j} \\ -1 & \text{if branch no. i goes from node j} \\ 0 & \text{otherwhise} \end{cases}$$

The power balance for the network in figure 1 is

$$\begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix}P_1\\P_2\\P_3\\P_4\\P_5\\P_6\end{pmatrix} = \begin{pmatrix}0\\0\\P_{D3}\end{pmatrix} \qquad (3)$$

The power flow P_6 is a fictive generator delivering power to node no. 3.

B. Transmission limitations

The transmission limitations in the different branches can be described by:

$$\mathbf{P}_{\min} \le \mathbf{P} < \mathbf{P}_{\max} \qquad (4)$$

where

- $\mathbf{P}_{\min}$: Lower power limit vector, normally 0 [kW]
- $\mathbf{P}_{\max}$: Upper power limit vector[kW].

C. Branch failures

Examples of branch failures are:

- Branch failure caused by independent failure in the components.
- Branch failure caused by common mode failure, which results in several component failures at the same time.
- Branch failure caused by an abnormal state in the surroundings like lightning or bad weather.

In addition, demand exceeding the delivery capacity may result in failure.

A branch having a failure has a reduced ability or is not able to transport power. This situation can be modeled by

$$\mathbf{P}_{\max,i} = \mathbf{P}_{\max f,i} \qquad (5)$$

where

- $\mathbf{P}_{\max,i}$: Maximal power limit in branch no. i [kW]
- $\mathbf{P}_{\max f,i}$: Maximal power limit in branch no. i in case of failure [kW].

The power limit in case of failure is normally $\mathbf{P}_{\max f,i} = 0$.

D. Calculation of power flow and power shortage

The power flow and power shortage is calculated by minimizing the cost function, equation (2), subject to the power flow balance, equation (1), the constraints defined by the power limitations, equation (4), and the branch failure limitations, equation (5). A power shortage has occurred if the fictive power $P_F > 0$.

III. Basic probability theory

A. The Markov model

The Markov model is a general method to describe the probability that a system is in a function state defined by the function state vector $\boldsymbol{\xi}$. The model has the following form [3], [4].

$$\dot{\mathbf{p}} = \mathbf{A}\mathbf{p} \qquad (6)$$

where

- $\mathbf{p}$: Probability vector describing the probability to stay in the function states $\boldsymbol{\xi}$
- $\dot{\mathbf{p}}$: Rate of change of $\mathbf{p}$ [1/year]
- $\mathbf{A}$: Transition rate matrix [1/year].

An additional requirement to the model is $\sum \mathbf{p} = 1$.

Dynamic solution of the differential equation (6) requires a starting value for the probability vector **p**. The stationary solution is independent of the starting value.

The residence times and departure frequencies for the different states are [3], [4]:

$$\boldsymbol{\theta} = diag\left(\mathbf{A}\right)^{-1} \qquad (7)$$

$$\mathbf{v} = diag\left(\boldsymbol{\theta}\right)^{-1}\mathbf{p} = diag\left(\mathbf{A}\right)\mathbf{p} \qquad (8)$$

where

- $\boldsymbol{\theta}$: Residence time vector [year]
- $\mathbf{v}$: Departure frequency vector [1/year].

The residence time is the mean duration time spent in a state and the departure frequency is the expected number of departures from the state each year.

B. Unit models

Complex systems may have a large number of unit components. The reliability models describing these units shall be called unit models. The unit models are assumed to have function state vectors $\boldsymbol{\xi}_j$ and state probability vectors defined by the Markov models

$$\dot{\mathbf{p}}_j = \mathbf{A}_j \mathbf{p}_j \tag{9}$$

The index j is the model identification. The unit models should generally have low dimension and be easy to define.

C. Composite models

Assume that the probability model for the system composed of the unit components, having function state vector $\boldsymbol{\xi}$, is defined by the Markov model (6). This composite model generally has a very large dimension, which makes the transition rate matrix impossible to establish, and the Markov equation impossible to solve. An alternative method to calculate the probability variables is the following formal functions:

$$\boldsymbol{\xi} = \Phi_{\xi}\left(\xi_1, \xi_2, \cdots \xi_\kappa\right) \tag{10}$$

$$\mathbf{p} = \Phi_p\left(\mathbf{p}_1, \mathbf{p}_2, \cdots \mathbf{p}_\kappa\right) \tag{11}$$

$$\mathbf{A} = \Phi_A\left(\mathbf{A}_1, \mathbf{A}_2, \cdots \mathbf{A}_\kappa\right) \tag{12}$$

where

- $\boldsymbol{\xi}$: Composite function state vector
- $\mathbf{p}$: Composite probability vector
- $\mathbf{A}$: Composite transition rate matrix
- ξ_j : Function state vector for unit model no. j
- $\mathbf{p}_j$: Probability vector for unit model no. j
- $\mathbf{A}_j$: Transition rate matrix for unit model no. j
- κ : Number of unit models.

The function $\Phi_p(\cdot)$ is based on Kronecker products and $\Phi_A(\cdot)$ on Kronecker sums [6], [7].

The function $\Phi_{\xi}(\cdot)$ represents a combination of the unit model function states. This is illustrated in the following example.

Assume a system consisting of two unit models with the function state vectors

$$\boldsymbol{\xi}_1 = \begin{Bmatrix} \xi_{1,1} \\ \xi_{1,2} \end{Bmatrix}; \quad \boldsymbol{\xi}_2 = \begin{Bmatrix} \xi_{2,1} \\ \xi_{2,2} \end{Bmatrix} \tag{13}$$

A composite function state vector is defined by

$$\boldsymbol{\xi} = \begin{Bmatrix} \xi_{1,1} \cap \xi_{2,1} \\ \xi_{1,1} \cap \xi_{2,2} \\ \xi_{2,1} \cap \xi_{2,1} \\ \xi_{2,1} \cap \xi_{2,2} \end{Bmatrix} \tag{14}$$

where $\cap$ is the logical *and* operator. However, for calculation purposes it is more convenient to operate with the function state index matrix Ψ defined by

$$\Psi_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \quad \Psi_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{15}$$

The composite function state index matrix is

$$\Psi = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{pmatrix} \tag{16}$$

In general the following relation can be defined

$$\Psi = \Phi_{\Psi}\left(\Psi_1, \Psi_2, \cdots \Psi_\kappa\right) \tag{17}$$

where

- Ψ : Function state index matrix for the composite model
- Ψ_j : Function state index matrix for the unit model no. j
- $\Phi_{\Psi}(\cdot)$: A function that generates the function state index matrix.

A compact formulation of the composite functions comprising equation (10),(11),(12) and (17) is

$$\left[\boldsymbol{\xi}, \Psi, \mathbf{p}, \mathbf{A}\right] = \Phi\left(\boldsymbol{\xi}_j, \mathbf{p}_j, \mathbf{A}_i\right)_{j=1\to\kappa} \tag{18}$$

D. Aggregated models

High dimension function state and probability vectors are difficult to interpret. More insight is obtained by aggregation. Typical aggregated function states are:

- The system is functioning
- The system has failed.

Aggregation of the probability vector can be defined by

$$\mathbf{p}_a = \mathbf{D}\mathbf{p} \tag{19}$$

where

- $\mathbf{p}_a$: Aggregated probability vector
- $\mathbf{p}$: Composite probability vector
- $\mathbf{D}$: Aggregation matrix.

The elements in **D** belong to the set $\{0, 1\}$. An additional requirement is that $\sum \mathbf{p}_a = 1$. This requirement is satisfied if the column sums of **D** are 1 for each column.

The function states of the aggregated model can formally be defined by

$$\boldsymbol{\xi}_a = \mathbf{D} \cup \boldsymbol{\xi} \tag{20}$$

where

- $\boldsymbol{\xi}_a$: Function state vector for the aggregated model
- $\boldsymbol{\xi}$: Function state vector for the composite model
- $\mathbf{D}$: The aggregation matrix interpreted as a logical matrix
- $\cup$: The logical *or* operator.

The aggregated transition rate matrix is formally defined by

$$\mathbf{A}_a = \Gamma_A\left(\mathbf{p}, \mathbf{A}, \mathbf{D}\right) \tag{21}$$

where

- $\mathbf{A}_a$: Transition rate matrix for the aggregated system
- $\mathbf{A}$: Transition rate matrix for the composite system
- $\mathbf{D}$: The aggregation matrix
- $\mathbf{p}$: Probability vector for the composite model.

The function $\Gamma_A(\cdot)$ is a matrix function performing the aggregation.

A compact formulation of the aggregation functions comprising equation (19), (20) and (21) is

$$\left[\boldsymbol{\xi}_a, \mathbf{p}_a, \mathbf{A}_a\right] = \Gamma\left(\boldsymbol{\xi}, \mathbf{p}, \mathbf{A}, \mathbf{D}_a\right) \tag{22}$$

Note that the aggregated probability variables ($\mathbf{A}_a$, $\mathbf{p}_a$) satisfies the Markov model. Therefore also the residence time and departure frequency vectors can be calculated for the aggregated model by relation (7) and (8).

IV. Power demand model

The demand of supply can be deterministic or stochastic. In case of stochastic demand a vector of demand levels $\boldsymbol{\xi}_D$ and a belonging probability vector can be used. The probability model can be a unit Markov model given by:

$$\dot{\mathbf{p}}_D = \mathbf{A}_D \mathbf{p}_D \tag{23}$$

where

$\mathbf{p}_D$: Probability for the different demand levels $\boldsymbol{\xi}_D$

$\mathbf{A}_D$: Transition rate matrix.

To avoid complex models the number of levels has to be kept as low as possible.

A composite probability vector including the network and the demand probabilities can be calculated in accordance with the principles used in the previous section.

V. Reliability models for the power Transmission network

A. Model structure

Failure in the transmission network can be described by:

- Branch failures
- Delivery failures.

Branch failures comprise all failures that reduce the ability of a branch to operate at maximum capacity. Failure in a branch may be caused by failure in one or more components that constitute the branch function.

Delivery failure represents shortage in the ability to deliver the demanded power. Owing to redundancy, a branch failure may not cause a delivery failure.

A function state vector can be used to describe the operating condition of each system function or component. The simplest possible function state vector for a component has the two following states:

- Functioning
- Failed.

The function state for the complete network is a result of the component function states, the network system structure, the network capacity and demand. To obtain all function states for a network, all combinations of component states must be evaluated. The dimension of the network function state vector is therefore growing very fast with the number of components. By using several function levels it is possible to partly overcome this problem. Typical function levels are:

- The unit model level
- The branch level
- The demand level
- The network level.

Power supply shortage is associated with the branch level. Probability models concerning these levels are discussed in the following paragraphs.

B. Unit component models

The probability for staying in a function state for a component in the network can be described by unit models. The overall network probability can then be obtained by combining the results from each unit model. The unit models are characterized by:

- A function state vector that describes the function states.
- A probability vector that describes the probability for staying in the different function states
- A model of the Markov type that describes the relation for calculation of the probability vector.

Typical unit models for a branch in an electrical grid is:

- Power lines or transformers
- Protection functions
- Automatic re-closing
- Common power supply
- Demand
- Environment impacts like whether or lightning.

The probability for the different unit model function states $\boldsymbol{\xi}_{c,ij}$ is based on the Markov models defined by:

$$\dot{\mathbf{p}}_{c,ij} = \mathbf{A}_{c,ij}\mathbf{p}_{c,ij} \tag{24}$$

where

$\boldsymbol{\xi}_{c,ij}$: Function state vectors

$\mathbf{p}_{c,ij}$: Probability vectors

$\mathbf{A}_{c,ij}$: Transition rate matrices.

ij : Index for unit model no. j in branch no. i.

All these models should have a low dimension.

C. Branch models

The branch probability variables are obtained by assembling the unit models into composite models. The result based on the composite function (18) is:

$$\left[\boldsymbol{\xi}_{b,i}, \Psi_{b,i}, \mathbf{p}_{b,i}, \mathbf{A}_{b,i}\right] = \Phi\left(\boldsymbol{\xi}_{c,ij}, \mathbf{p}_{c,ij}, \mathbf{A}_{c,ij}\right)_{j=1\to\alpha_i} \tag{25}$$

where

$\xi_{b,i}$: Function state vector for branch no. i

$\Psi_{b,i}$: Function state index matrix for branch no. i

$\mathbf{p}_{b,i}$: Probability vector for branch no i

$\mathbf{A}_{b,i}$: Transition rate matrix for branch no i.

α_i : Number of unit models in branch no. i.

Aggregation of these composite probability variables based on the aggregation function (22) gives

$$\left[\boldsymbol{\xi}_{ba,i}, \mathbf{p}_{ba,i}, \mathbf{A}_{ba,i}\right] = \Gamma\left(\boldsymbol{\xi}_{b,i}, \mathbf{p}_{b,i}, \mathbf{A}_{b,i}, \mathbf{D}_{a,i}\right) \tag{26}$$

where

$\boldsymbol{\xi}_{ba,i}$: Aggregated function state vector for branch no. i

$\mathbf{p}_{ba,i}$: Aggregated probability vector for branch no. i

$\mathbf{A}_{ba,i}$: Aggregated transition rate matrix for branch no. i

$\mathbf{D}_{b,i}$: Aggregation matrix for branch no. i

The aggregated branch variables ($\mathbf{A}_{ba,i}$, $\mathbf{p}_{ba,i}$) satisfy the Markov model. Therefore, also the residence time vector $\boldsymbol{\theta}_{b,i}$ and the departure frequency vector $\mathbf{v}_{b,i}$ for the branches can be calculated according to equation (7) and (8).

Several levels of aggregation can be used to evaluate the branch probability. For evaluation of the network functionality the following branch function states $\boldsymbol{\xi}_{ba,i}$ are used:

- The branch is functioning
- The branch is failed.

Based on these function states the network reliability models can be composed.

D. Network models

In this case the purpose is to analyze the probability for a failure in the network. The function states for the network are obtained by combination of all states in the aggregated branch models. The result based on the composite function (18) is:

$$\left[\xi_n, \Psi_n, \mathbf{p}_n, \mathbf{A}_n\right] = \Phi\left(\xi_{ba,i}, \mathbf{p}_{ba,i}, \mathbf{A}_{ba,i}\right)_{i=1\to\beta} \tag{27}$$

where

ξ_n : Function state vector

Ψ_n : Function state index matrix

$\mathbf{p}_n$: Probability vector

$\mathbf{A}_n$: The transition rate matrix

β : Number of branches.

The function states are then separated in functioning states and failure states and a belonging aggregation matrix is generated. The resulting aggregated model according to the aggregation function (22) is

$$\left[\xi_{na}, \mathbf{p}_{na}, \mathbf{A}_{na}\right] = \Gamma\left(\xi_n, \mathbf{p}_n, \mathbf{A}_n, \mathbf{D}_n\right) \tag{28}$$

where

ξ_{na} : Function state vector

$\mathbf{p}_{na}$: Probability vector

$\mathbf{A}_{na}$: Transition rate matrix.

$\mathbf{D}_n$: Aggregation matrix

Also the probability variables ($\mathbf{A}_{na}$, $\mathbf{p}_{na}$) satisfy the Markov model. The residence time vector $\boldsymbol{\theta}_{na}$ and the departure frequency vector $\boldsymbol{\nu}_{na}$ for the network can therefore be calculated according to equation (7) and (8).

E. Power supply reliability

In this case the purpose is to analyze the power delivery shortage. The analysis is based on the network function states obtained by all possible combinations of the branch function states {Functioning, Failed}. In the case of stochastic demand these states are also combined with the network states.

The power delivery shortage is calculated by running the power flow model with the branch conditions defined by the network states. In case of fictive power delivery, a power supply shortage has occurred. The network probability variables are calculated by the composite function (18). The result is contained in the following set of variables

$$\left[\xi_{np}, \mathbf{A}_{np}, \mathbf{p}_{np}, \mathbf{P}_{np}\right] \tag{29}$$

where

ξ_{np} : Function state vector

$\mathbf{A}_{np}$: Transition rate matrix

$\mathbf{p}_{np}$: The probability vector

$\mathbf{P}_{np}$: Power supply shortage matrix.

The power supply shortage matrix has one column for each load node.

The expected power supply shortage is defined by

$$\hat{\mathbf{P}}_{np} = \mathbf{P}_{np} \bullet \mathbf{p}_{np} \tag{30}$$

where

$\hat{\mathbf{P}}_{np}$: Expected power supply shortage matrix

$\bullet$: Operator for row elements by vector element multiplication.

Expected power supply shortage for each node is further calculated by

$$\hat{\mathbf{P}}_{npc} = \Sigma_C \hat{\mathbf{P}}_{np} \tag{31}$$

where

$\hat{\mathbf{P}}_{npc}$: Line vector containing expected power shortage for each node.

Σ_C : Column sum operator.

The total expected power supply shortage is

$$\hat{P}_{npt} = \Sigma_R \hat{\mathbf{P}}_{npc} \tag{32}$$

where

$\hat{P}_{npt}$: Total expected power shortage

Σ_R : Row sum operator.

The function state vector ξ_{np} can be separated into the sets {No power supply shortage, Power supply shortage} Based on these sets an aggregation matrix can be generated. The aggregated probability variables are then calculated according to the aggregation function (22). The result is

$$\left[\xi_{npa}, \mathbf{p}_{npa}, \mathbf{A}_{npa}\right] = \Gamma\left(\xi_{np}, \mathbf{p}_{np}, \mathbf{A}_{np}, \mathbf{D}_{np}\right) \tag{33}$$

where

ξ_{npa} : Aggregated function state vector

$\mathbf{p}_{npa}$: Aggregated probability vector

$\mathbf{A}_{npa}$: Aggregated transition rate matrix

$\mathbf{D}_{npa}$: Aggregation matrix.

Again the residence time vector $\boldsymbol{\theta}_{npa}$ and the departure frequency vector $\boldsymbol{\nu}_{npa}$ can be calculated from $\mathbf{A}_{npa}$ according to equation (7) and (8).

VI. ECONOMIC MODEL

The economic consequence of a power supply shortage is

$$v_{ns} = c_{ns} \hat{P}_{npt} \tag{34}$$

where

v_{ns} : Cost flow for power not supplied [NOK/year]

c_{ns} : Specific cost for power not supplied [NOK/kW year]

$\hat{P}_{npt}$: Total power supply shortage [kW].

In a life cycle analysis the following economic balance equation can be used

$$\begin{aligned} &\dot{V} = rV + \sum v_{in,i} - \sum v_{out,i} \\ &V(0) = 0 \end{aligned} \tag{35}$$

where

V : Accumulated capital [NOK]

$V(0)$: Start value

$v_{in,i}$: Economic in-flow variables [NOK/year]

$v_{out,i}$: Economic out-flow variables [NOK/year]

r : Continuous rate of interest [1/year].

The cost flow for power not supplied v_{ns} is a part of the economic out-flows. For analysis of reliability consequences, equation (35) can be simplified by removing all flows not dependent on the reliability. This equation together with the probability model can be used for long-term operation and investment optimization.

VII. Implementation

The theory is preliminary implemented in a MATLAB program and tested for the network shown in figure 1. The input data is organized in the following groups:

- Unit models reliability data
- Network data
- Economic data.

A lot of reliability calculations concerning unit models, branch models and network models are available. Typical variables are:

- Probability for the different function states
- Residence time for the different function states
- Departure frequency for the different function states
- Expected power shortage
- Expected power shortage cost.

At the moment the reliability calculations are based on the exponential distribution. However, other distributions as the Weibull distribution seem to be easy to implement in the unit models. Both stationary and dynamic analysis can be performed. A special function is used to identify the transition rate matrix for demand models based on recorded data.

VIII. Results

The calculations are very fast and give a good insight in the network reliability. Here, only the expected cost of energy not served as a function of demand is shown in figure 2. The specific cost of energy not served is 39.84 [NOK/kW h] or 348998[NOK/kW year]

The curve "Non selective disconnection" represents the case in which the network breaks down when the demand is greater than the delivery capacity.

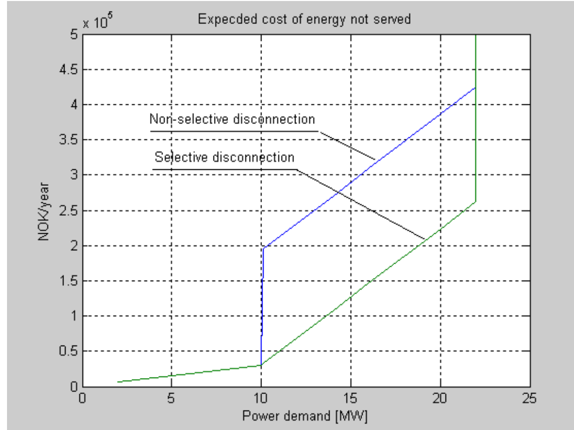

Fig. 2. Expected cost of energy not served

IX. Conclusion

The method seems to be very efficient especially for large systems. The unit model based method together with the model composition and aggregation may lead to an extreme reduction in manual work and calculation time. Extension of the practical dimension limits for Markov models from about 100 to almost infinite is possible. In the power grid described in this paper the dimension for a complete conventional Markov model is near $6*10^6$, a dimension that is completely unpractical. The method described in this paper uses 17 small models easy to establish and require just a few seconds of calculation time. Problems regarding modeling of common mode failure and failures caused by the surroundings are almost absent. Yet, no comparisons with other methods have been investigated. Comparisons with conventional Markov models are not relevant. However, comparison with Monte Carlo methods should be of interest.

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